12 Kinetic Theory

A significant amount of pressure is exerted by the paint molecules on the body of the can in which it is contained. When the top of the can is pressed, the volume inside the can gets reduced and the paint is thrown out with great pressure. Since the pressure has an inverse relationship with the volume, the application of the Boyle's law can be observed here.

Topic Notes

- *Kinetic Theory and Other Properties of Gases*

KINETIC THEORY AND OTHER PROPERTIES OF GASES

1

TOPIC 1

BEHAVIOUR OF PERFECT GAS AND KINETIC THEORY OF GASES

Molecular nature of matter

The atomic hypothesis has been given by many scientists. According to this hypothesis, everything in this universe is made up of atoms. Atoms are small particles that move in a permanent order and attract each other when they are small distance apart. But when they are forced to be very close to each other, they repel each other.

Dalton's atomic theory is also known as the molecular theory of matter. This theory proves this problem that gases are made up of molecules and molecules are made up of atoms.

According to Gay Lussac's law, when gases chemically bond to form another gas, their volumes are in the ratio of small integers.

Avogadro's law states that the same amount of gases at the same temperature and pressure have a same number of molecules.

Example 1.1: Why do atoms of different elements differ in mass?

Ans. Each and every element has a different mass. Force, carbon's molecular weight is 12.0107u. Oxygen's molecular weight is 15.999u and nitrogen's molecular weight is 14.0067u.

Hence, it's different for different elements.

TOPIC 2

PRESSURE AND BEHAVIOUR OF GASES

Boyle's Law

It is also known as Marriott's law. It is the relationship between the compression and expression of a gas at a constant temperature. This empirical relationship, formulated by scientist Robert Boyle in 1662, shows that the pressure (P) of a given amount of gas changes inversely with its volume (V) at a given temperature.

This relationship was also discovered by the French physicist Edme Monotte (1676). The law can be driven from the kinetic theory of gas assuming a perfect (ideal) gas. Real gas follows Boyle's law at a sufficiently low pressure but in general, at high pressures where the gas begins to deviate from its pleat behaviour, the product PV decreases slightly. Mathematically,

$$V \propto \frac{1}{P} \text{ or } V = \frac{k}{P} \text{ or } PV = k$$

Where, k is constant.

Its value depends on:

(1) mass of the gas

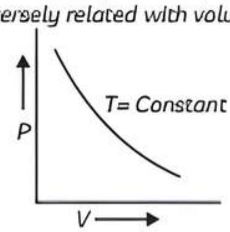
(2) its temperature

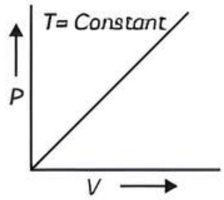
(3) the units in which P and V are measured.

Important

➥ *Students must know that according to Boyle's law, at a constant temperature, pressure of a given mass of gas varies inversely related with volume.*

➥





If P_1 and V_1 are the initial values of pressure and volume and P_2 and V_2 are their final values, then according to Boyle's law

$$P_1 V_1 = P_2 V_2$$

Example 1.2: At a temperature of 40°C, what will be the minimum pressure required to compress a 600 dm³ of gas at 2 bar to 800 dm³? [NCERT]

Ans. Given,

$P_1 = 2$ bar

$P_2 = ?$

$V_1 = 600 \text{ dm}^3$

$V_2 = 300 \text{ dm}^3$

Boyle's law state that:

Initial pressure × Initial volume

= Final pressure × Final volume

i.e., $P_1 V_1 = P_2 V_2$

$2 \times 600 = P_2 \times 300$

$$P_2 = \frac{(2 \times 600)}{300}$$
$$= \frac{1200}{300}$$
$$= 4 \text{ bar}$$

So, the minimum pressure required to compress the 300 dm^3, the volume of gas is 4 bar.

Charles's Law

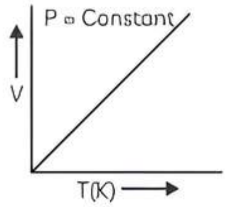
This law describes the relationship between the temperature and volume of a gas at constant pressure. It was discovered in the year 1707 by Alexander Charles. It states that if the pressure remains constant, then the volume of a given mass of a gas increases or decreases by $\frac{1}{273.15}$ of its volume at 0°C for 1°C rise or fall of temperature.

$$V_t = V_0 \frac{T}{T_0},$$

Or $$\frac{V_t}{T} = \frac{V_0}{T_0}$$

Or $$\frac{V}{T} = \text{constant}$$

i.e. $$V \propto T$$



So, Charle's law can be stated in another way.

Important

➥ *Pressure remains constant and the volume of a great mass of gas is directly proportional to its absolute temperature.*

Caution

➥ *Students must know that the Charle's law is a special case of the ideal gas law. The law is applicable to the ideal gases that are held at constant pressure but the temperature and volume keep changing.*

Example 1.3: **Find the initial volume of a gas at 150 K, if the final volume is 6 L at 100 K. [NCERT]**

Ans.
$V_1 = ?$
$V_2 = 6$ L
$T_1 = 150$ K
$T_2 = 100$ K

Using Charle's law,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{V_1}{(150)} = \frac{6}{100}$$

$$V_1 = \frac{6 \times 150}{100} = 9 \text{ L}$$

The initial volume of a gas at 150 K is 9 L.

Ideal Gas

A gas which obeys the ideal gas equation.

$$PV = nRT$$

At all temperatures and pressure, the above equation is called an ideal gas or perfect gas.

While deriving the ideal gas equation, the following two assumptions are used.

(1) The size of the gas molecules is negligibly small.

(2) There is no force of attraction amongst the molecules of the gas.

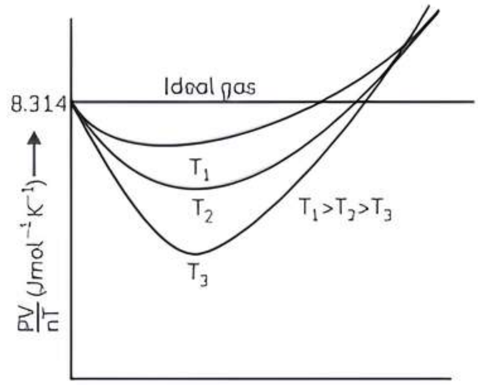
However, no real or actual gas fulfils the above conditions.

Hence, the behaviour of a real gas differs from that of an ideal gas. At low pressure and high temperature, the above assumptions are valid and real games like hydrogen, oxygen, nitrogen, helium etc almost behave like an ideal gas.

Deviation from ideal behaviour:

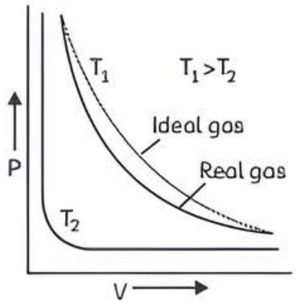
(1) The graph shows of $\frac{PV}{nT}$ against pressure P for three different temperatures for an ideal gas.

$$\frac{PV}{nT} = R = 8.314 \text{ Jmol}^{-1} \text{ K}^{-1}.$$

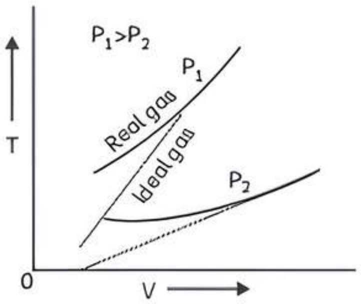


Clearly, departures from ideal gas behaviour becomes less at low pressures and high temperature.

(2) The graph shows the comparison between the experimental P-V curves and the theoretical curves, predicated by the Boyle's law.



(3) The graph shows the comparison between experimental T-V curves and the theoretical curves predicted by Charle's law.



Important

➥ *In the above cases, we note the behaviour of a real gas approaches the ideal gas behaviour for low pressure and high temperature.*

Example 1.4: Using the ideal gas equation, determine the value of R. Given that, 1 g molecule of a gas at S.T.P occupies 22.4 L. [NCERT]

Ans. Here,

$$P = 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$
$$T = 273 \text{ K}$$
$$V = 22.4 \text{ L} = 22.4 \times 10^{-3} \text{ m}^3$$

For 1 mole of a gas, PV = RT

$$\therefore \quad R = \frac{PV}{T}$$
$$= \frac{1.013 \times 10^5 \times 22.4 \times 10^{-3}}{273}$$
$$= 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

TOPIC 3

KINETIC THEORY OF AN IDEAL GAS

The kinetic theory of gas suggests that the gas is composed of moving particles. Pressure is applied by the continuous impact of the gas on each surface. The higher, the density of the gas, the more often the molecules collide with the surface and the greater the pressure applied. Therefore, as the volume of a particular mass of gas decreases and the pressure increases more, gas is pumped into the container.

At the rise of the temperature, the velocity of the molecule increases, increasing both the number and momentum given by each collision. This explains the increase in gas pressure as the temperature rises.

Assumptions

(1) All gases consist of molecules. The molecules are rigid, elastic spheres, identical in all respects for a given gas and different for different gases.

(2) The size of a molecule is negligible compared with the average distance between the molecules.

(3) The molecules are in a state of continuous random motion. Moving in all directions with possible velocities.

(4) During the random motion, the molecules collide with one another and with the walls of the vessels.

(5) The collisions are perfectly elastic and there are no forces of attraction or repulsion between the molecules. Thus, all internal energy of the gas is kinetic.

(6) Between two collisions, a molecule moves in a straight path with a uniform velocity. The average distance covered by a molecule between two successive collisions is called mean free path.

(7) The collisions are almost instantaneous *i.e.,* the time during which a collision lasts is negligible compared to the time of the free path between the molecules.

(8) In spite of the molecular collisions, the density remains uniform throughout the gas.

Kinetic Interpretation of Temperature

The pressure exerted by an ideal gas is given by,

$$P = \frac{1}{3}\frac{mNv^2}{V}$$

$$P = \frac{1}{3}\frac{\left(\frac{1}{2}mv^2\right)N}{V}$$

$$P = \frac{2}{3}\frac{E}{V}$$

$$E = \frac{3}{2}PV$$

$$E = \frac{3}{2}NK_BT$$

$$\frac{E}{N} = \frac{3}{2}K_BT$$

So, we can see that the average kinetic energy of a molecule is proportional to the absolute temperature of the gas. It is independent of pressure, volume or the nature of the ideal gas.

This is a fundamental result relating to the temperature to the internal energy of a molecule. This is a kinetic interpretation of temperature.

Root Mean Square Speed (v_{rms})

The *rms* speed (root mean square speed) is defined as the square root of the mean of the squares, of the random speeds of the individual molecules of a gas.

Assume, random speed of the molecules of gas are $v_1, v_2, v_3, ... v_n$.

$$v_{rms} = \left[\frac{v_1^2 + v_2^2 + v_3^2 + ... + v_n^2}{n}\right]^{1/2}$$

According to the kinetic theory.
Pressure of an ideal gas,

$$P = \frac{1}{3}\frac{mNv^2}{V}$$

$$3PV = mN_A v^2 \quad (\text{if, } N = N_A)$$

$$v^2 = \frac{3PV}{M}$$

$$\text{Because, } v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$= \sqrt{\frac{3K_BT}{m}}$$

Where,
M: mass of one mole of gas.
m: Mass of one molecule of gas.

Average Speed (v_{av})

It is the arithmetic mean of the speed of molecules of a given gas at a given temperature.

$$v_{av} = \frac{|v_1|+|v_2|+\ldots}{N}$$

and according to the kinetic theory,

$$v_{av} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8K_BT}{\pi m}}$$

$$= \sqrt{\frac{8}{3\pi}}v_{rms} = 0.92\ v_{rms}$$

Where,

M: mass of one mole of gas

m: Mass of one molecule of gas

Most Probable Speed (v_{mp})

It is the speed possessed by maximum number of molecules in a given gas at a given temperature.

$$v_{mp} = \frac{\sqrt{2RT}}{M} = \frac{\sqrt{2K_BT}}{m}$$

$$= 0.82\ v_{rms}$$

TOPIC 4

LAW OF EQUIPARTITION OF ENERGY

It states that in any dynamical system in thermal equilibrium, the energy is equally amongst its various degree of freedom and the energy associated with each degree of freedom per molecule is $\frac{1}{2}K_BT$.

Where, K_B is Boltzmann's constant and T is the absolute temperature of the system.

Consider one mole of a monatomic gas in thermal equilibrium at temperature T. A monatomic gas molecule can be taken as a point mass. So each such molecule has 3 degrees of freedom due to translatory motion.

According to the kinetic theory of gases, the average translational K.E. of a gas molecule is given by,

$$\frac{1}{2}mv^2_{rms} = \frac{3}{2}K_BT$$

Where, $\overrightarrow{v^2_{rms}}$ is the mean square velocity of a gas molecule of mass m. If $\vec{v_x^2}, \vec{v_y^2}$, and $\vec{v_z^2}$ are the components of mean square velocity of the gas molecules along the three-coordinate axis,

then,

$$\vec{v^2} = \vec{v_x^2} + \vec{v_y^2} + \vec{v_z^2}$$

$$\frac{1}{2}m\vec{v_x^2} + \frac{1}{2}m\vec{v_y^2} + \frac{1}{2}m\vec{v_z^2} = \frac{3}{2}K_BT$$

As the molecular motion is random, there is no preferred direction of motion. So, the average kinetic energy of each molecule along each of the three axes is the same.

$$\frac{1}{2}m\vec{v_x^2} = \frac{1}{2}m\vec{v_y^2} = \frac{1}{2}m\vec{v_z^2}$$

Coming from the above two equations,

we get,

$$\frac{1}{2}m\vec{v_x^2} = \frac{1}{2}m\vec{v_y^2} = \frac{1}{2}m\vec{v_z^2} = \frac{1}{2}K_BT$$

Thus, the average K.E. per molecule per degree of freedom is $\frac{1}{2}K_BT$. This result was first deduced by Boltzmann and is called the law of equipartition of energy.

Important

➥ *The law of equipartition of energy holds goods for all degrees of freedom whether translational, rational or vibrational.*

Example 1.5: Case Based:

You can observe a real-life application of Boyle's Law when you fill your bike tires with air. When you pump air into a tire, the gas molecules inside the tire get compressed and packed closer together. This increases the pressure of the gas, and it starts to push against the walls of the tire.

(A) Assertion (A): If a gas container in motion is suddenly stopped, the temperature of the gas rises.

Reason (R): The kinetic energy of ordered mechanical motion is converted into the kinetic energy of random motion of gas molecules.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

[Delhi Gov. QB 2022]

(B) If $K_B = 1.38 \times 10^{-23}$ J molecule^{-1} K^{-1} mass per molecule of the gas = 6.4×10^{-27} kg, the K.E. per molecule of a gas at 127°C will be:

(a) 8.28×10^{-21} J (b) 6.28×10^{-21} J
(c) 10.28×10^{-21} J (d) 7.28×10^{-21} J

(C) According to the kinetic theory, what is the average kinetic energy of molecules of an ideal gas?

(D) The average kinetic energy of $Br_{(g)}$ compared to $Cl_{2(g)}$, at the same temperature, would be greater. Justify.

(E) Use the kinetic molecular theory, to explain, why the pressure changes, when the temperature of an ideal gas increases.

Ans. (A) *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: The motion of the container is known as the ordered motion of the gas and zigzag motion of gas molecules within the container is called disordered motion. When the container suddenly stops, ordered kinetic energy gets converted into disordered kinetic energy which increases the temperature of the gas.

(B) *(a) 8.28×10^{-21} J*

Explanation: Here,

$$T = 127 + 273 = 400 \text{ K}$$
$$m = 6.4 \times 10^{-27} \text{ kg}$$

K.E. per molecule,

$$= \frac{1}{2} mv^2_{rms} = \frac{3}{2} K_B T$$
$$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 400$$
$$= 8.28 \times 10^{-21} \text{ J}$$

(C) The average kinetic energy of a molecule is proportional to the absolute temperature of the gas. It is independent of pressure, volume or the nature of the ideal gas and given by, $E = \frac{S}{2} K_b T$.

(D) The *KE* of a single gas particle can be described in terms of mass *m* and velocity *v*.

$$K.E. = \frac{(mv^2)}{2}$$

If the gases have the same temperature, then their respective K.E. values will vary according to their respective molar masses. Diatomic bromine has a higher molar mass than diatomic chlorine. Therefore, the average K.E. of bromine must be higher than the average kinetic energy of chlorine at the same temperature.

(E) According to kinetic molecular theory, the pressure of a gas is due to the collision of gas molecules to the walls of the container. The more particles and the faster they hit the walls result in higher pressure. When temperature increases, the K.E. of the gas molecules also increases. This means that they are moving faster. Since the molecules hit the walls of the container faster (indicating more collisions of the gas molecules with the walls of the container) which result in an increase in the pressure of the gas.

Example 1.6: A mixture of 1 mole of oxygen and 2 mole of nitrogen at 300 K is stored in a vessel. The ratio of the average rotational kinetic energy of O_2 a molecule to that of N_2 molecules per degrees of freedom is:

(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 2 : 2 [NCERT]

Ans. (a) 1 : 1

Explanation: According to the law of equipartition of energy.

Energy associated with each molecule for each degree of freedom, $E = \frac{1}{2}$ KT

Both N_2 and O_2 are diatomic gas.

∴ Rational degree of freedom, $(f_r) = 2$ for both N_2 and O_2

∴ Average rotational kinetic energy per molecule, $E_r = \frac{1}{2}KT$

Therefore, average rotational K.E. associated with O_2 molecule per degree of freedom = average rotational K.E. associated with N_2 molecule per degree of freedom.

TOPIC 5

SPECIFIC HEAT CAPACITY

Monatomic Gases

There are a number of monatomic gases like He, Ar, etc. All the molecules have three translational degrees of freedom.

According to the law of equipartition of energy, average energy associated with each degree of freedom per molecule $= \frac{1}{2}K_BT$

Average energy associated with three degrees of freedom per molecule $= \frac{3}{2}K_BT$

Let R = Gas constant per mole of a gas

N_A = Avogadro's number

Then, the total internal energy of 1 mole of a monatomic gas,

$$U = \frac{3}{2}K_BT \times N_A = \frac{3}{2}RT$$

$[K_B N_A = R]$

Molar specific heat at constant volume,

$$C_v \text{ (monoatomic)} = \frac{dU}{dT} = \frac{d}{dT}\left(\frac{3}{2}RT\right) = \frac{3}{2}R$$

Molar specific heat at constant pressure,

$$C_P \text{ (monoatomic)} = C_V + R$$

$$= \frac{3}{2}R + R = \frac{5}{2}R$$

Specific heat ratio,

$$\gamma = \frac{C_P}{C_V} = \frac{\left(\frac{5}{2}\right)R}{\left(\frac{3}{2}\right)R}$$

$$= \frac{5}{3} = 1.67$$

Diatomic Gases

(1) The behaviour of diatomic molecules such as N_2, O_2 etc are rigid rotators at moderate temperatures. Such molecules have 5 degrees of freedom: 3 translational and 2 rotational. According to the law of equipartition of energy the total energy of a mole of as:

$$U = \frac{5}{2}K_BT \times N_A = \frac{5}{2}RT$$

$$\therefore C_v \text{ (rigid diatomic)} = \frac{dU}{dT} = \frac{5}{2}R$$

$$C_P \text{ (rigid diatomic)} = C_V + R = \frac{7}{2}R$$

Specific heat ration,

$$\gamma \text{ (rigid diatomic)} = \frac{\left(\frac{7}{2}\right)R}{\left(\frac{5}{2}\right)R} = \frac{7}{5} = 1.4$$

(2) If the diatomic molecule is not rigid but has also a vibrational mode, then each molecule has an additional equal to $2 \times \left(\frac{1}{2}\right) K_B T = K_B T$.

because a vibrational frequency has both kinetic and potential energy modes.

∴

$$U = \left(\frac{5}{2}K_BT \times K_B\right)$$

$$= \frac{7}{2}K_BN_AT = \frac{7}{2}RT$$

C_V (diatomic with vibrational mode)

$$= \frac{dU}{dT} = \frac{7}{2}R$$

C_P (diatomic with vibrational mode)

$$= C_v + R$$

$$= \frac{9}{2}R$$

Specific heat ratio,

$$\gamma = \frac{\left(\frac{9}{2}\right)R}{\left(\frac{7}{2}\right)R} = \frac{9}{7} = 1.28$$

Polyatomic Gases

Consider, 1 mole of a perfect polyatomic gas at absolute temperature T. Suppose the total degrees of freedom of each molecule be f. According to the law of equipartition of energy, According, energy of each molecule is $\frac{f}{2}K_BT$

$\therefore$ Internal energy of one mole of a gas

$$U = \frac{f}{2}K_BT \times N_A = \frac{f}{2}RT$$

$$C_V = \frac{dU}{dT} = \frac{f}{2}R$$

$$C_P = C_V + R$$

$$= \frac{f}{2}R + R = \left(\frac{f}{2}+1\right)R$$

$$r = \frac{C_P}{C_V} = \frac{\left(\frac{f}{2}+1\right)}{\frac{f}{2}R}$$

or $$r = 1 + \frac{2}{f}$$

Specific Heats of Solid (Dulong and Petit's law)

Near the room temperature, the molar specific heat of most of the solids at constant volume is equal to 3R or 6 cal mol^{-1} K^{-1} or 25J mol^{-1} K^{-1}. This statement is known as Dulong and Petit's law.

In a solid state, atoms are present in mean positions.

When vibration occurs, K.E. (E_K) of an atom continuously changes into P.E. (E_P) and vice versa.

So, the average value E_K and E_P are equal in the solid state.

Since, atom can vibrate along three mutually perpendicular directions, it has 3 degrees of freedom.

Apply the law of equipartition of energy,

$$E_K = 3 \times \frac{1}{2}K_BT = \frac{3}{2}K_BT$$

$$E_P = 3 \times \frac{1}{2}K_BT = \frac{3}{2}K_BT$$

$\therefore$ Average vibrational energy per atom,

$$E_K + E_P = 3K_B T$$

Total vibrational energy or the internal energy of 1 mole of atoms of the solid is gives by,

$$U = N_A \times 3K_B T = 3RT$$

$$[R = K_B N_A]$$

Since for a solid, V is negligible, so

$$\Delta Q = \Delta U + P\Delta V \cong \Delta U$$

$\therefore$ $$C_V = \frac{\Delta Q}{\Delta T} = \frac{\Delta U}{\Delta T} = 3R$$

This proves the Duolong and Petit's law.

Important

➥ *Specific heats and molar specific heats of some solids at room temperature and atmospheric pressure.*

Substance	*Specific heat ($J\ kg^{-1}\ K^{-1}$)*	*Molar specific heat ($J\ mol^{-1}\ K^{-1}$)*
Aluminium	*900.0*	*24.4*
Carbon	*506.5*	*6.1*
Copper	*386.4*	*24.5*
Lead	*127.7*	*26.5*
Silver	*236.1*	*25.5*
Tungsten	*134.4*	*24.9*

Example 1.7: According to Dalton's atomic theory, matter consists of indivisible ....

(a) molecules **(b) atoms**

(c) ions **(d) mixtures** **[NCERT]**

Ans. *(b) atoms*

Explanation:

Molecules— a group of atoms bonded together, representing the smallest fundamental unit of a chemical compound that can take part in a chemical reaction.

Atoms— Atoms are the basic unit of life. A molecule is a compound made up of two or more atoms held by chemical bonds. The mixture is a combination of pure substances in a ratio. Ion is either positively or negatively charged.

Ions— An atom or molecule with a net electric charge due to the loss or gain of one or more electrons.

Mixtures— A combination of different things in which the component elements are individually distinct.

Example 1.8: If one mole of polyatomic gas has two vibrational modes and β is the ratio of molar specific heats for polyatomic gas $\beta = \frac{C_P}{C_V}$, then what is the value of β? **[NCERT]**

Ans. Here, number of translational degree of freedom = 3

Number of rotational degree of freedom = 3

Number of vibrational degree of freedom

$= 2 \times 2 = 4$

Total number of degree of freedom

$= 3 + 3 + 4 = 10$

Degree of freedom of polyatomic gas

$$\beta = \frac{C_P}{C_V} = 1+\left(\frac{2}{f}\right) = 1+\left(\frac{2}{10}\right)$$

$$\beta = \frac{12}{10} = 1.2$$

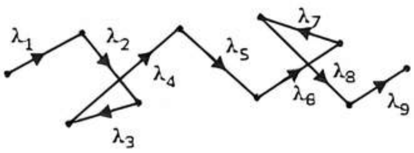
TOPIC 6

MEAN FREE PATH

The molecules of a gas are in a state of continuous, rapid and random motion. As these molecules have a finite though small size, so they collide against one another frequently.

Between two successive collisions, a molecule moves along a straight line path with uniform velocity. It is also known as free path.

But after every collision, velocity of each molecule changes both in magnitude and direction. Hence, each molecule follows a series of straight lines like *zig-zag* paths.



The mean free path of a gas molecule may be defined as the average distance travelled by the molecule between two successive collisions. As shown in the given figure, if a molecule covers free paths $\lambda_1, \lambda_2, \lambda_3$... after successive collision, then its mean free path is given by,

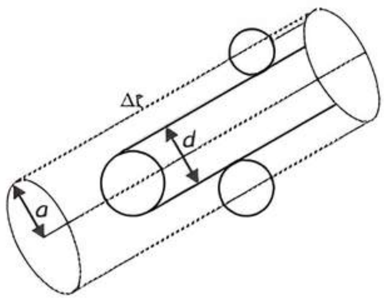
$$\lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 + ...}{\text{Total number of collisions}}$$

Expression: In order to derive the expression for the mean free path, we make use of the following assumptions:

(1) Each molecule of the gas is a sphere of diameter *d*.

(2) All molecules of the gas except the molecule A has speed *v*.

It will collide with all those molecules, whose centre lies within distance *d* from its path.



In Δt time, it will collide with all the molecules in the cylinder of volume $\pi d^2\ v\Delta t$. Let *n* be the number of molecules per unit volume

Number of collisions suffered by the molecule *A* in time Δt = Volume of the cylinder swept by molecule *A* in time Δt × Number of molecules per unit volume.

$$= \pi d^2 v\Delta t \times n$$

Mean free path of gas molecule (when molecules are at rest position),

$$\lambda = \frac{\text{Distance covered in time } \Delta t}{\text{Number of collisions suffered in time } \Delta t}$$

$$= \frac{v\Delta t}{\pi d^2 v\Delta t\, n} = \frac{1}{n\pi d^2}$$

When the gas molecules are in motion,

$$\lambda = \frac{1}{\sqrt{2}n\pi d^2}$$

If *m* is the mass of gas molecules, then the density of the gas is,

$$\rho = mn$$

or $$n = \frac{\rho}{m}$$

∴ $$\lambda = \frac{m}{\sqrt{2}\pi d^2\rho}$$

For 1 mole of gas,

$$PV = RT$$

Or $$P = \frac{RT}{V}$$

$$= \frac{N}{V} \times \frac{R}{N} nK_BT$$

Or $$n = \frac{P}{K_BT}$$

∴ $$\lambda = \frac{K_BT}{\sqrt{2}\pi d^2 p}$$

Important

➥ *Mean free path is directly proportional to the mass of the gas molecule.*

➥ $\lambda \propto \frac{1}{p}$, *the mean free path is inversely proportional to the pressure of the gas.*

Example 1.9: The mean free path, having an average speed 20 m/s and collision frequency 10 s^{-1} is:

(a) 1 m (b) 2 m
(c) 3 m (d) 0.1 m [NCERT]

Ans. *(b) 2 m*

Explanation: Collision frequency is the number of times, a molecule of a gas collides with other molecules.

We know,

Distance = Speed × Time

Hence,

$$\text{Mean free path} = \frac{\text{Speed}}{\text{Frequency}}$$

$$= \frac{20\ m/s}{10\ s^{-1}} = 2\ m$$

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. At what temperature is the *rms* velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C?

(a) –73 K (b) 3 K

(c) 20 K (d) 80 K

[Delhi Gov. QB 2022]

Ans. *(c) 20 K*

Explanation:

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

Now, *rms* velocity of H_2 molecule *rms* velocity of O_2 molecule

$$\sqrt{\frac{3R \times T}{2}} = \sqrt{\frac{3R \times (47+273)}{32}}$$

$$T = \frac{2 \times 320}{32} = 20\,K$$

2. The amount of energy that must be supplied to a solid substance in order to trigger a change in its physical state and convert it into a liquid is known as:

(a) Latent heat of fusion

(b) Evaporation

(c) Solidification

(d) Specific latent heat

Ans. *(a) Latent heat of fusion*

Explanation: Energy supplied to convert a unit mass of a substance from a solid to a liquid state at its melting point is called latent heat of fusion. Evaporation is a form of vaporisation that usually happens on the surface of liquids, and it involves the transition of the liquid into the gaseous phase. Solidification is a process in which atoms are converted into an ordered solid state from a liquid disordered state. Specific latent heat is defined as the amount of thermal energy (heat *Q*) that is absorbed or released when a body undergoes a content temperature pressure.

Caution

➥ *Students must know that Charle's law is a special case of the ideal gas law. The law is applicable to the ideal gases that are held at constant pressure but the temperature and volume keep changing.*

3. The Latent heat of 6 kg substance, if the amount of heat for a phase change is 300 kcal is:

(a) 50 kcal (b) 60 kcal

(c) 40 kcal (d) 30 kcal

Ans. *(a) 50 kcal*

Explanation: Given that:

$$Q = 300 \text{ kcal}$$
$$M = 6 \text{ kg}$$

The formula for latent heat is given by,

$$L = \frac{Q}{M}$$

$$L = \frac{300}{6}$$

$$L = 50 \text{ kcal}$$

4. A piece of helium has a volume of 420 mL at 100°C. The temperature at which the volume will become 240 mL is:

(Assume that the pressure is constant)

(a) –86.5°C (b) –76.4°C

(c) –59.85°C (d) –50.23°C

Ans. *(c) –59.85°C*

Explanation: Given that:

$$V_1 = 420 \text{ mL}$$
$$V_2 = 240 \text{ mL}$$
$$T_1 = 100 + 273 = 373 \text{ K}$$
$$T_2 = ?$$

Since, pressure remains constant.

Therefore, By applying Charles's law,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Or $$T_2 = \left(\frac{T_1}{V_1}\right) \times V_2$$

$$T_2 = \left(\frac{373}{420}\right) \times 240 = 213.1\text{K}$$

Or $$T_2 = 213.1 - 273 = -59.85°\text{C}$$

5. The work done by (or on) a gas per mole per Kelvin is called:

(a) Universal gas constant

(b) Boltzmann's constant

(c) Gravitational constant

(d) Entropy [Delhi Gov. QB 2022]

Ans. *(a) Universal gas constant*

Explanation: $\frac{PV}{T} = R$

6. Gas exerts with a volume of 7 dm^3 and a pressure of 3.03 × 10^5 kPa. And the gas is completely stimulated into another tank where it exerts a pressure of 2.02 × 10^5 kPa

at the same temperature. The volume of the tank is:

(a) 10 dm^2 (b) 10.5 dm^3
(c) 9 dm^3 (d) 5 dm^3

Ans. *(b) 10.5 dm^3*

Explanation: Given:

Initial volume, $V_1 = 7\ dm^3$

Initial pressure, $P_1 = 3.03 \times 10^5$ kPa

Final pressure, $P_2 = 2.02 \times 10^5$ kPa

Since temperature is constant.

Final volume $(V_2) = ?$

At constant temperature,

By using Boyle's law

$$P_1V_1 = P_2V_2$$

$$V_2 = \frac{P_1V_2}{P_2}$$

$$V_2 = \frac{3.03\times10^5 \times 7\ dm^3 \times kpa}{2.02\times10^5\ kPa}$$

$$V_2 = 10.5\ dm^3$$

Hence, the volume of the tank is 10.5 dm^3

⚠ Caution

➥ *Students should know that at high temperatures and low pressure, modules are far apart. When the temperature is increased, the molecules will move randomly far from each other.*

7. **The RMS velocity of molecules of a gas of which the ratio of two specific heats is 3.42 and velocity of sound in the gas is 500 m/s is:**

(a) 727 m/s **(b) 580 m/s**
(c) 468 m/s **(d) 390 m/s**

Ans. *(c) 468 m/s*

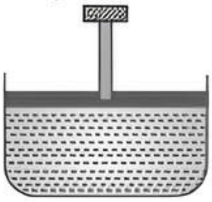
Explanation:

$$v_s = \sqrt{\frac{3.42}{3}}$$

$$\Rightarrow \quad 500 = \sqrt{1.14}\, v_{rms}$$

$$v_{rms} = \frac{500}{\sqrt{1.14}} = 468 \text{ m/s.}$$

8. **A cylinder containing an ideal gas in a vertical position and has a piston of mass M that is able to move up or down without friction. If the temperature is increased:**



(a) both P and V of the gas will change.

(b) only P will increase according to Charle's law.

(c) V will change but not P.

(d) P will change but not V.

[NCERT Exemplar]

Ans. *(c) V will change but not P*

Explanation: If P_{atm} is the atmospheric pressure acting on top of the piston and A is the cross area of the piston itself, then the total pressure acting on the gas molecules below, is equal to

$$P = \frac{M_g}{A}\ Q\ \frac{M_r}{A}:$$

$$= \frac{\text{Force due to piston of mass M}}{A}$$

$$= \text{Pressure}$$

As atmospheric pressure is constant, so the value of M, g and A, the pressure due to the piston, is also constant for the system.

At constant pressure, when temperature is increased, the volume of the gas increases (the piston moves up).

9. **A vessel of volume V contains a mixture 1 mole of hydrogen and 1 mole of oxygen (both considered as an ideal gas). Let f_1 = (v)dv, denote the fraction of molecules with speed between v and (v) + dv similarly for oxygen. Then:**

(a) f_1 (v) + f_2 (v) = f(v) obeys the Maxwell's distribution law.

(b) f_1 (v) + f_2 (v) will obey the Maxwell's distribution law separately.

(c) Neither f_1 (v) nor f_2 (v) will obey the Maxwell's distribution law.

(d) f_2 (v) and f_1 (v) will be the same.

[NCERT Exemplar]

Ans. *(b) f_1 (v), f_2 (v) obey the Maxwell's distribution law separately.*

Explanation: The Maxwell's speed distribution law is given as follows,

$$\frac{dN}{dv} = 4\pi N\left(\frac{m}{2\pi KT}\right)^{\frac{3}{2}} v_o^2 - \left(\frac{mv^2}{2KT}\right)$$

Where, $dN = f(v)$ represents the number of molecules having velocities between v and $v + dv$.

K is the Boltzmann constant.

T is the absolute temperature.

m is the mass of the ideal gas used.

v is the velocity of the ideal gas.

From the above relation, we conclude that the Maxwell-Boltzmann speed distribution function $\left(\frac{dN}{dv}\right)$ is proportional to the mass of the gas used and since the molecular mass of hydrogen and that of oxygen are different. So, the two gases f_1 (v) and f_2 (v) would obey Maxwell's law separately.

10. A flask contains argon and chlorine in the ratio of 2 : 1 by mass. The temperature of the mixture is 27°C. The ratio of average kinetic energies of two gases per molecule is:

(a) 1 : 1 (b) 2 : 1

(c) 3 : 1 (d) 6 : 1 [Diksha]

Ans. *(a) 1 : 1*

Explanation: The average translational kinetic energy (per molecule) of any ideal gas (be it monatomic like argon, diatomic like chlorine or polyatomic is always equal to $\frac{3}{2}K_B T$. It depends only on temperature and is independent of the nature of the gas.

Since argon and chlorine, both have the same temperature in the flask, the ratio of average kinetic energy (per molecule) of the two gases is 1 : 1.

Caution

➥ *Students must know that if ρ = density of gas.*

Then, $\lambda \propto \frac{1}{\rho}$, i.e., mean free path is inversely proportional to the density of the gas.

11. A gas filled in a closed vessel is heated through 1 K and its pressure increases by 0.4%. What was the initial temperature of the gas?

(a) 250 K (b) 350 K

(c) 450 K (d) 500 K

[Delhi Gov. QB 2022]

Ans. *(a) 250 K*

Explanation: Volume of constant.

$$\therefore \quad P \propto T$$

$$\therefore \quad \frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$\frac{P}{P+0.004P} = \frac{T+273}{T+274}$$

$$\therefore \quad \frac{1}{1.004} = \frac{T+273}{T+274}$$

$$\therefore \quad T = -23°C$$

$$\therefore \quad T = 250 \text{ K}$$

Assertion Reason Questions

Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

12. Assertion (A): Bus is driven by a driver at the same time air pressure increases.

Reason (R): Temperature of the air in the tyre increases due to friction of the tyre with the road. Increase in temperature results in an increase in pressure according to Charle's law.

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: As per Charle's law, since the volume of the tyre remains constant. Hence, $P \propto T$ and thus, the pressure of gas in the tyre increases.

13. Assertion (A): **The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume.**

Reason (R): **The molecules of a gas collide with each other and the velocities of the molecules change due to collision.**

[Delhi Gov. QB 2022]

Ans. *(b) Both A and R are true and R is not the correct explanation of A.*

Explanation: Total translational kinetic energy

$$= \frac{3}{2}nRT = \frac{3}{2}PV$$

In an ideal gas all molecules moving randomly in all directions collide and their velocity changes after the collision.

The molecules are said to move in a chaotic (random) manner since they are never in a steady state. They move in a straight line in every direction while moving at varying but consistent speeds. When molecules collide with a container or with other molecules, their motion changes in both direction and velocity.

14. Assertion (A): The ratio of specific heat of gas to the constant pressure of diatomic gas and constant volume of specific heat is more than that of the monatomic gas.

Reason (R): Degrees of freedom has more molecules of monoatomic gas than the number of diatomic gas molecules

Ans. *(d) A is false and R is also false.*

Explanation: As we know,

$$\frac{C_P}{C_V} = r = 1 + \frac{2}{f}$$

Where, f is the degree of freedom.

Now, for a monatomic gas, $f = 3$

So, $r_m = 1 + \frac{2}{3} = \frac{5}{3} = 1.6$

For a diatomic gas, $f = 5$

So, $r_d = 1 + \frac{2}{5} = \frac{7}{5} = 1.4$

Thus, $r_d < r_m$

but $f_m < f_d$

15. Assertion (A): **The mean free path of gas molecules changes in inverse proportion to the density of the gas.**

Reason (R): **The mean free path of a gas molecule is defined as the mean distance that the molecule travels between two consecutive collisions.**

Ans. *(b) Both A and R are true and R is not the correct explanation of A.*

Explanation: The mean free path of a gas molecule is the average distance between two successive collisions. It is represented by λ.

$$\lambda = \frac{1}{\sqrt{2}}\frac{KT}{\pi^2 P} = \frac{m}{\sqrt{2\pi^2 \sigma d}}$$

Here, σ = 0 diameter of a molecule

and K = Boltzmann's constant

$\Rightarrow \qquad \lambda \propto \frac{1}{d'}$

$\Rightarrow \qquad \lambda \propto T$

and $\qquad \lambda \propto \frac{1}{P}$

Hence, the mean free path varies inversely with the density of the gas. It can easily prove that the mean free path varies directly with the temperature and varies as the pressure of the gas varies.

⚠ Caution

↪ *Students should know that each square term in the total energy expression of a molecule contribution towards one degree of freedom.*

16. Assertion (A): **Equal masses of helium and oxygen gases are given equal quantities of heat. There will be a greater rise in the temperature of helium compared to that of oxygen.**

Reason (R): **The molecular weight of oxygen is more than the molecular weight of helium.**

[Delhi Gov. QB 2022]

Ans. *(b) Both A and R are true and R is not the correct explanation of A.*

Explanation: There will be a greater rise in temperature of helium compared to oxygen because helium has a greater specific heat capacity.

Also, the molecular weight of oxygen is 16 and that of helium is 4.

CASE BASED Questions (CBQs)

[**4** & **5** marks]

Read the following passages and answer the questions that follow:

17. *According to this law, for any system in thermal equilibrium, the total energy is equally distributed among its various degree of freedom. And each degree of freedom is associated with energy 1/2 KT. (where K = 1.3 × 10^{-23} J/K, T = absolute temperature of the system).*

At a given temperature T; all ideal gas molecules no matter what their mass have the same average translational kinetic energy; namely, 3/2 KT.

When measure the temperature of a gas, we are also measuring the average translational kinetic energy of its molecules.

At the same temperature, gases with different degrees of freedom (e.g. He and H) will have different average energy or internal energy namely f/2 KT (f is different for different gases).

[Delhi Gov. QB 2022]

(A) Relation between pressure P and average kinetic energy E per unit volume of a gas is:

(a) $P = \frac{2E}{3}$ (b) $P = \frac{E}{3}$

(c) $P = \frac{3E}{2}$ (d) $P = 3E$

(B) At 0 K, which of the following properties of a gas will be zero?

(a) Kinetic energy

(b) Potential energy

(c) Vibrational energy

(d) Density

(C) The root mean square velocity of a gas molecule of mass *m* at a given temperature is proportional to:

(a) m^0 (b) m

(c) $\sqrt{m}$ (d) $m^{-\frac{1}{2}}$

(D) An ant is walking on the horizontal surface. The number of degrees of freedom of an ant will be:

(a) 1 (b) 2

(c) 3 (d) 6

(E) The number of degrees of freedom for a diatomic gas molecule is:

(a) 2 (b) 3

(c) 5 (d) 6

Ans. (A) *(a)* $P = \frac{2E}{3}$

Explanation:

$$\text{Kinetic Energy} = \frac{3RT}{2}$$

$$\Rightarrow \quad \text{K.E.} = \frac{3PV}{2}$$

Since, PV = nRT, here n = 1 mole

Therefore, $P = \frac{2E}{3V}$, (E = Kinetic Energy)

As per question,

$$\frac{E}{V} = E \quad \text{(As volume = 1)}$$

So, $P = \frac{2E}{3}$

(B) *(a) Kinetic Energy*

Explanation: *At 0 K, all molecular motion stops, so kinetic energy becomes zero.*

(C) *(d)* $m^{-1/2}$

Explanation: $V_{rms} = \sqrt{\frac{3K_BT}{m}}$, *i.e.* $v_{rms} \propto m^{-\frac{1}{2}}$

(D) *(b) 2*

Explanation: As the ant can move on a plane, it has 2 degrees of freedom.

(E) *(c) 5*

Explanation: A diatomic molecule has 3 degrees of freedom due to translatory motion and 2 degrees of freedom due to rotatory motion.

18. *Cooking utensils are made of metal which has low specific heat capacity so that they need less heat to raise up the temperature. Handles of cooking utensils are made of substances with high specific heat capacities so that their temperature won't become too high even if it a absorbs large amount of heat.*

(A) A gas occupies a volume of 400 cm^3 at 0°C and 780 mm of Hg. How many litres of volume will the gas occupy at 80 degree Celsius and 780 mm of Hg?

(B) At very low pressure and high temperature, the real gas behaves like an ideal gas. Why?

(C) A tank of volume 0.3 m^3 contains 2 moles of Helium gas at 200°C. Assuming the helium behaves as an ideal gas, what will be the total internal energy of the system?

Ans. (A) According to the question,

$$V_1 = 400 \text{ cm}^3$$

$$T_1 = 0°\text{C} = 0 + 273 = 273 \text{ K}$$

$$T_2 = 80°\text{C} = 80 + 273 = 353 \text{ K}$$

You need to find V_2.

Here, only the temperature is changing, the pressure remains constant.

Using Charle's law,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Putting the above values in the Charle's law, we get,

$$\frac{400}{273} = \frac{V_2}{353}$$

$$V_2 = 400 \times \frac{353}{273}$$

$$V_2 = 517.21 \text{ cm}^3$$

Since, 1 cm^3 = 0.001 litres,

Then, 517.21 cubic cm

$$= 517.21 \times 10^3$$

$$= 0.517 \text{ L}$$

(B) An ideal gas is one which has zero volume of molecule and no intermolecular forces. Now:

(1) At very low pressure, the volume of gas is large, so that the volume of molecule is negligible compared to volume of gas.

(2) At very high temperatures, the kinetic energy of molecules is very large and effect of intermolecular forces can be neglected.

Hence, real gases behave as an

ideal gas at low pressure and high temperature.

(C) n = No. of moles = 2

T = Temperature = 273 + 20 = 293 K

R = Universal Gas constant = 8.31 J/mole/K

Total energy of the system $E = \frac{3}{2}nRT$

$$E = \frac{3}{2} \times n \times 8.31 \times 293$$

$$E = 7.30 \times 10^3 \text{ J}$$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

19. Two molecules of gas have speeds of 9×10^6 m/s, and 1×10^6 m/s respectively. What is the root mean square speed of these molecules? [NCERT Exemplar]

Ans. Given that: Speed of first gas molecules, $v_1 = 9 \times 10^6$ m/s

Speed of second gas molecules, $v_2 = 1 \times 10^6$ m/s

The root mean square speed of the gas molecules is equal to

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2}{2}}$$

$$v_{rms} = \sqrt{\frac{(9\times10^6)^2 + (1\times10^6)^2}{2}} \text{ m/s}$$

$$v_{rms} = \sqrt{\frac{81\times10^{12} + 1\times10^{12}}{2}} \text{ m/s}$$

$$v_{rms} = \sqrt{\frac{(81+1)\times10^{12}}{2}} \text{ m/s}$$

$$v_{rms} = \sqrt{41}\times10^6 \text{ m/s}$$

$$v_{rms} = 6.40 \times 10^6 \text{ m/s}$$

20. A container has an equal number of molecules of hydrogen and carbon dioxide. If a fine hole is made in the container, then which of the two gases shall leak out rapidly? [Delhi Gov. QB 2022]

Ans. Hydrogen (Because *rms* speed is greater).

21. In a closed container a gas is heated to 2°C, and its pressure increases by 0.6%. Evaluate the initial temperature of the gas.

Ans. Given that: $P = \frac{0.6}{100}.P$

$$T = T + 2$$

By Gay lussac's law,

$$\frac{P}{T} = \frac{\left(\frac{P+0.6}{100.P}\right)}{T+2}$$

$$\frac{(P+0.006P)}{(T+2)} = \frac{P(1.006)}{(T+2)}$$

$$T + 2 = (1.006)T$$

$$1 = 0.006T$$

$$T = 166.66 \text{ K}$$

22. There are two gases named hydrogen and oxygen which are stored in a box. If there is a tiny hole in the box, which gas will discharge very fastly? Why?

Ans. We know that $v_{rms} \propto \frac{1}{\sqrt{M_0}}$

Hence, hydrogen gas will leak more rapidly of its smaller molecular mass.

23. The absolute temperature of a gas is increased 3 times. What is the effect on the root mean square velocity of the molecules? [Delhi Gov. QB 2022]

Ans. Given that:

Absolute temperature of the gas is increased 3 times.

$$T_2 = 3T_1$$

Root mean square velocity is given by,

$v_{rms} = \sqrt{\frac{3RT}{M}}$, where R is the gas constant, T is the temperature in Kelvin, and M is the molecular weight of the gas.

So, from the above equation, $V_{rms} \propto \sqrt{T}$

$$\Rightarrow \frac{V_{rms_1}}{V_{rms_2}} = \sqrt{\frac{T_1}{T_2}}$$

$$\Rightarrow V_{rms_2} = \left(\sqrt{\frac{T_2}{T_1}}\right)V_{rms_1}$$

$$\Rightarrow V_{rms_2} = \left(\sqrt{\frac{3T_1}{T_1}}\right)V_{rms_1}$$

$$\Rightarrow V_{rms_2} = \left(\sqrt{3}\right)V_{rms_1}$$

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

24. A gas storage tank has a small leak. The pressure in the tank drops more quickly if the gas is hydrogen than if it is oxygen. Why?

[Delhi Gov. QB 2022]

Ans. The square root of the density has an inverse relationship with the rate of gas diffusion. Hydrogen escaped more quickly as a result.

25. The volume of the air bubbles increases 15 times when it rises from the bottom of the lake to the top. Calculate the depth of the lake, if the density of the lake water is 1.02×10^3 kg.m^3 and atmospheric pressure is 75 cm of Hg.

Ans. Here, $P_1 = 75$ cm of Hg

$= 0.75 \times 13.6 \times 10^3 \times 9.8 \text{ Nm}^{-2}$

$= 99.96 \times 10^3 \text{ Nm}^{-2}$

Let V_2 = volume of bubble at depth $h = x$

i.e., $V_1 = x + 15x = 16x$

$P_2 = 75$ cm of Hg + $h\rho$ water g

$= 99.96 \times 10^3 + h \times 10^3 \times 9.8$

Using the Boyle's law

$P_1V_1 = P_2V_2$

We get $99.96 \times 10^2 + 16x$

$= (99.96 \times 10^3 + h \times 10^3 \times 9.8)x$

Or $h = \frac{15 \times 99.96 \times 10^3}{9.8 \times 10^3}$

$= 153$ m

26. Estimate the total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other consistent) in a room of a volume capacity 25.0 m^3 at a temperature of 27°C and 1 atm pressure. The Boltzmann constant K is 1.38×10^{23} JK^{-1} and atm is 1.01×10^5 Nm^{-2}

Ans. For a given mass of gas containing n molecules, the gas equation is

$PV = nKT$

$\therefore \; n = \frac{PV}{KT}$

Here, $P = 1 \text{ atm} = 1.01 \times 10^5 \text{ Nm}^{-2}$,

$V = 25.0 \text{ m}^3$

$K = 1.38 \times 10^{-23} \text{ JK}^{-1}$

$= 1.38 \times 10^{-23} \text{ NmK}^{-1}$

$T = 27°C = 27 + 273 = 300$ K

$$n = \frac{(1.01 \times 10^5 \text{ Nm}^{-1}) \times 25.0 \text{ m}^3}{(1.38 \times 10^{-23} \text{ NmK}^{-1}) \times 300 \text{ K}}$$

$= 6.10 \times 10^{26}$

27. Evaluate K.E. of one mole of gas at normal temperature and pressure. What will be its value at 273°C ? (R = 8.21 mol^{-1}s^{-1})

Ans. According to the kinetic theory, (the K.E. of 1 mole of an ideal gas is given by, $E = \frac{3}{2}RT$.)

At normal temperature, T = 0°C = 273 K

$\therefore \; E = \frac{2}{2} \times 8.31 \text{ J mol}^{-1} \text{K}^{-1} \times 273 \text{ K}$

$= 3.40 \times 10^3 \text{ J mol}^{-1}$

The K.E. of a gas is directly proportional to its absolute temperature.

$\therefore \; \frac{E_{273}}{E_0} = \frac{273 + 273}{273} = 2$

$E_{273} = E_0 \times 2$

$= (3.40 \times 10^3 \text{ J mol}^{-1}) \times 2$

$= 6.80 \times 10^3 \text{ J mol}^{-1}$

28. A gaseous mixture consists of 2.0 moles of oxygen and 4.0 moles of neon at temperature T. Neglecting all vibrational modes, calculate the total internal energy of the system. (Oxygen has two rotational modes). [NCERT Exemplar]

Ans. Oxygen is a diatomic molecule, *i.e.* a molecule of oxygen has two atoms of oxygen. So, it has 5 degrees of freedom (3 due to translational motion and 2 due to rotational motion). The total energy of 2 moles of oxygen is,

$$E_1 = \frac{5}{2} \times 2 \times RT$$

$$= 5RT$$

Similarly, neon is monatomic (contains a single atom) and thus has 3 degrees of freedom, outgoing to its translational motion only.

So, total energy of 4 moles of neon gas is,

$$E_2 = \frac{3}{2} \times 4 \times RT$$

$$= 6RT$$

The total internal energy of the gaseous mixture is equal to,

$$E = E_1 + E_2$$

$$= 5RT + 6RT$$

$$= 11RT$$

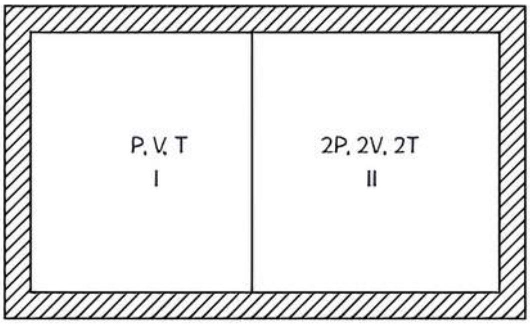
29. A gas is filled in a cylinder fitted with a piston at a definite temperature and pressure. Why the pressure of the gas decreases when the piston is pulled out? [Delhi Gov. QB 2022]

Ans. As a result, fewer molecules will collide with the cylinder's walls each second, transferring less momentum to the walls as a result. Furthermore, the bigger area of the walls is where these impacts occur. The pressure drops for each of these causes.

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

30. A partition divides a container having insulated walls into two compartments I and II. The same gas fills the two compartments. What is the ratio of the number of molecules in compartments I and II?



[Delhi Gov. QB 2022]

Ans. The pressure in the two compartments will be equal because the wall has to be at rest and it is not accelerating.

Let the volume on the left side is x and the volume on the right side will be

$3V - x$ and the final pressure in compartment I is P_1.

The expression for the left side is given as,

$$P_1 \times x = PV \quad \text{...(i)}$$

The expression for the left side is given as,

$$P_1 \times (3V - x) = 2P \times 2V \quad \text{...(ii)}$$

Dividing equation (i) by equation (ii), we get

$$(3V - x) = 4x$$

$$x = \frac{3V}{5}$$

The volume of the second compartment is,

$$3V - x = 3V - \frac{3V}{5}$$

$$= \frac{12V}{5}$$

The final pressure is given as,

$$P_1 \times \frac{3V}{5} = PV$$

$$P_1 = \frac{5P}{3}$$

Thus, the pressure in the two compartments are equal, the volume in compartment I is $\frac{3V}{5}$, the volume in compartment II is $\frac{12V}{5}$ and the final pressure in compartment I is $\frac{5P}{3}$.

31. There are n gas molecules in the contains. If you increase the number of molecules to $2n$, what will be:

(A) Gas pressure?

(B) Total gas energy?

(C) RMS rate of gas molecules? [Diksha]

Ans. (A) We know that,

$$P = \frac{1}{3} mnc^2$$

Where, n = number of molecules per unit volume

Thus, when number of molecules is increased from n to $2n$, number of molecules per unit volume (n) will increase from n to $2n$.

(B) The K.E. of a gas molecule is,

$$\frac{1}{2}mc^2 = \frac{3}{2}KT$$

If the number of molecules is increased from n to $2n$. There is no effect on the average K.E. of a gas molecule, but the total energy is doubled.

(C) RMS speed of the gas is,

$$v_{rms} = \sqrt{\frac{3P}{P}} = \sqrt{\frac{3P}{mn}}$$

When n is increased from n to $2n$, both n and P become double and the ratio $\frac{P}{n}$ remains unchanged. So, there will be no effect of increasing the number of molecules from n to $2n$ on *rms* speed of gas molecules.

32. We have 0.5 g of hydrogen gas in a cubic chamber of size 3 cm kept at NPT. The gas in the chamber is compressed, keeping the temperature constant till a final pressure of 100 atm. Is one justified in assuming the ideal gas law, in the final state?

(Hydrogen molecules can be considered as a sphere of radius 1 Å) [NCERT Exemplar]

Ans. Initial volume of the cubic gas chamber is equal to,

$$V_i = (3 \times 10^{-2})^3 \text{ m}^{-3}$$
$$= 27 \times 10^{-6} \text{ m}^{-3}$$

Volume of one molecule of hydrogen gas is given as,

$$V = \frac{4}{3} \times \pi \times (1 \times 10^{-10})\text{m}^{-3}$$

$$= \frac{4}{3} \times 3.14 \times 1 \times 10^{-10} \text{ m}^{-3}$$

$$= 4.186 \times 10^{-30} \text{ m}^{-3}$$

Total number of moles of hydrogen gas present in 0.5 g of it.

$$n = \frac{\text{Mass of hydrogen gas present}}{\text{Molar mass of hydrogen gas}}$$

$$n = \frac{0.5}{2} = 0.25$$

So, total number of molecules present in 0.25 moles of hydrogen gas.

$$N = n \times 6.023 \times 10^{23}$$
$$= 0.25 \times 6.023 \times 10^{23}$$
$$= 1.505 \times 10^{23}$$

So, total volume occupied by 1.505×10^{23} number of molecules of hydrogen gas.

$$V_{total} = (1.505 \times 10^{23} \times 4.186 \times 10^{-30} \text{ m}^{-3}$$
$$= 6.299 \times 10^{-7} \text{ m}^{-3}$$

Now, let P_f and V_f be the final pressure and final volume of the hydrogen as inside the chamber upon compression at a constant temperature. The gas obeys ideal gas laws.

then,

$$P_iV_i = P_fV_f$$

$$V_f = \left(\frac{P_i}{P_t}\right) \times V_i$$

$$= \frac{1}{100} \times 27 \times 10^{-6} \text{ m}^{-3}$$

$$= 27 \times 10^{-9} \text{ m}^{-3}$$

$$= 2.7 \times 10^{-7} \text{ m}^{-3}$$

We conclude, when a gas is compressed at a constant temperature, intermolecular forces come into play and the molecules deviate from ideal gas behaviour.

33. Ten light aircraft fly at a speed of 150 km/h in volume airspace in total darkness, 20 × 20 × 1.5 km³ you are on one of the planes and are randomly flying through the room without knowing where the other planes are. On average about how long it will collapse between near collisions with your aircraft. Assume for this rough computation that a safety region around the plane can be approximated by a sphere with a radius of 10 m.

Ans. The mean free path of the plane is given as,

$$\lambda = \frac{1}{\sqrt{2}\pi d^2}$$

Where, n is the number of aircraft per unit volume and is equal to, $n = \frac{10}{V}$

d is the diameter of the safety zone, *i.e.*,

$$d = 2r = 20 \text{ m}$$

If λ is the distance travelled between successive collisions then time elapsed t, between these two collisions can be given as,

$$t = \frac{\lambda}{v} = \frac{1}{\sqrt{2}\pi n d^2 v}$$

$$= \frac{v}{\sqrt{2}\pi N d^2 v}$$

$$t = \frac{20 \times 20 \times 1.5}{1.414 \times 3.14 \times 10 \times 20^2 \times 10^{-6} \times 150} \text{ hrs}$$

$$= \frac{1.5}{150} \times \frac{1}{1.414 \times 3.14} \times 10^5 \text{hrs}$$

$$= \frac{1}{1.414 \times 3.14} \times 10^5 \text{ hrs}$$

$$= 0.22522 \times 10^3 \text{ hrs}$$

$$= 225.22 \text{ hrs}$$

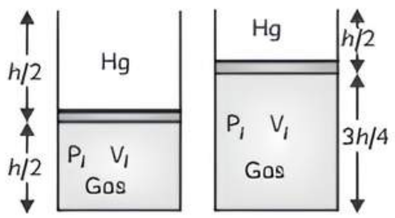
So, the amount of time elapsed between two near successive collisions of the aircraft is equal to 225.22 hrs.

34. The 1.52 m high vertical hollow cylinder is equipped with a movable piston with very little mass and thickness. The lower half of the cylinder contains the ideal gas and the upper half is filled with mercury. The cylinder is initially at 300 K. As the temperature rises, half of the mercury comes out of the cylinder. Calculate this temperature. Assuming that the thermal expansion of mercury is negligible (atmospheric pressure = 0.76 m Hg).

Ans. Let P_i, V_i, T_i be the initial pressure, volume and temperature of the gas when it occupies half the height $\frac{h}{2}$ of the cylinder. On heating, the gas expands to $\frac{3h}{4}$.

Let P_f, V_f, T_f be the final values of the parameter. For an ideal gas,

We have, $\frac{P_iV_i}{T_i} = \frac{P_fV_f}{T_f}$



Let P be the atmospheric pressure and ρ is the density of Hg.

Then $P_i = P + \frac{h}{2}\rho . g$

Where, $h = 1.52\text{m}$

$= (0.76 \text{ m})\rho g + \left(\frac{1.52}{2}\text{m}\right)$

ρg

$= (1.52 \text{ m})\rho g = 2\rho g$

$V_i = \frac{h}{2}A.$

Where, A is the cross-sectional area.

$T_i = 300 \text{ K}.$

$$P_f = P + \frac{h}{4}\rho g$$

$$= P + \left(\frac{1.52}{4}m\right)\rho g$$

$$= P + \frac{P}{2} = \frac{3P}{2}$$

and $V_f = \frac{3h}{4}A$

$$T_f = \frac{P_fV_f}{T_f} \times T_i$$

$$= \frac{(3P/2)(3h/4)}{(2P)(h/2)A} \times T_i$$

$$= \frac{9}{8}T_i = \frac{9}{8} \times 300\text{K}$$

$$T_f = 337.5 \text{ K}$$

35. Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monatomic) the second contains chlorine (diatomic) and the third contains uranium hexafluoride (polyatomic). Do the vessels contain an equal number of respective molecules? Is the root mean square speed of molecules the same in the three cases? If not in which case are v_{rms} the largest?

[Delhi Gov. QB 2022]

Ans. The three vessels are identical in terms of volume and capacity. As a result, the pressure, volume, and temperature of each gas are the same.

The three containers will each hold an equal amount of the different molecules, according to Avogadro's law. This quantity equals, $N = 6.023 \times 10^{23}$, the number discovered by Avogadro.

The root mean square speed (v_{rms}) of a gas of mass m, and temperature T, is given by the relation:

$$v_{rms} = \sqrt{\frac{3KT}{m}}.$$

Where,
K is Boltzmann constant
For the given gases, K and T are constants.
Hence, v_{rms} depends only on the mass of the atoms,

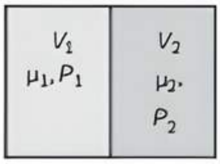
i.e., $v_{rms} \propto \left(\frac{1}{m}\right)^{\frac{1}{2}}$

As a result, the molecules' root mean square speed in the three scenarios differs. Neon has the least mass compared to uranium hexafluoride, chlorine, and chlorine. Thus, among the listed gases, neon has the highest root mean square speed.

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

36. **The container shown in fig have two chambers, separated by a partition of volume $V_1 = 2.0$ L and $V_2 = 3.0$ L. The chambers contain $\mu_1 = 4.0$ and $\mu_2 = 5.0$ moles of gas at pressure $P_1 = 1.00$ atm and $P_2 = 2.00$ atm. Calculate the pressure after the partition is removed and the mixture attains equilibrium.**



[NCERT Exemplar]

Ans. The ideal gas equation for the both gases in separate compartments before mixing can be written as:

$P_1V_1 = \mu_1 RT_1$ and $P_2V_2 = \mu_2 RT_2$

According to the kinetic theory of gases,

$$PV = \frac{2}{3}\mu E$$

Where, E is the internal energy of the gas.

So, before the two gases are mixed together, the total internal energy of the system will be equal to the sum of internal energy of each individual gas as follows;

$$E = \mu_1 E_1 + \mu_2 E_2$$

Where, μ_1 and μ_2 is the number of moles of each gas and E_1 and E_2 are the internal energy of one mole of each of the gases, respectively.

Substituting values,

We get, $$E = \frac{3}{2} P_1V_1 + \frac{3}{2} P_2V_2$$

Upon mixing the total number of moles of gas present is,

$$\mu = \mu_1 + \mu_2$$

Total volume of the system,

$$V = V_1 + V_2$$

Let the total pressure of the system, when equilibrium is reached, be P. Then, since the total internal energy of the system does not change on mixing, the energy of the mixture after mixing is equal to the total energy before mixing, i.e., E.

Then these are: $PV = \frac{2}{3}E$

$$P(V_1 + V_2) = \frac{2}{3} \times \{\frac{3}{2}(P_1 V_1 + P_2 V_2)\}$$

$$P = \frac{(P_1V_1 + P_2V_2)}{V_1 + V_2}$$

$$P = \frac{(1\times2+2\times3)}{2+3} \text{ atm}$$

$$P = \frac{2+6}{5} \text{ atm} = 1.6 \text{ atm}$$

37. **The temperature of an ideal gas is T K and the mean kinetic energy of its molecules is given by the following relation;**

$$E = 3.07\times10^{-23}\ T(\text{joule/K})/\text{molecule}$$

Calculate the number of molecules in one litre of the gas at N.T.P.

What will be the average distance between the molecules?

Ans. The mean kinetic energy of a gas molecule, at absolute temperature T, is given by,

$$E = \frac{3}{2} KT$$

Where, K is Boltzmann's constant.

Here, $E = 3.07 \times 10^{-23}$ T joule/K

$$\frac{3}{2} K = 3.07 \times 10^{-23} \text{ J/K}$$

$\therefore$ $$K = \frac{2}{3} \times (3.07\times10^{-23} \text{ J/K})$$

$$K = 2 \times 10^{-23} \text{ JK}^{-1}$$

There are N (Avogadro's number) molecules in 1 mole of the gas.

For this, the gas equation is,

$$PV = RT = NKT \qquad \left[K = \frac{R}{N}\right]$$

$\therefore$ If a given mass of the gas has n molecules, then, for this mass of the gas, the equation will be:

$$PV = nKT$$

$\therefore$ $$n = \frac{PV}{KT}$$

Here, $P = 1.01 \times 10^5$ Nm^{-2} (normal pressure),
$V = 1$ litre $= 1000$ cm$^3 = 10^{-3}$ m^3
and $T = 273$ K (normal temperature)

$\therefore$ $$n = \frac{(1.01\times10^{+5})\times10^{-3}}{(2\times10^{-23})\times273}$$

$$= 1.85 \times 10^{22}$$

Thus, there are 1.85×10^{22} molecules in a volume of 10^{-3} m^3.

Hence, space (average volume) available for one molecule is,

$$= \frac{10^{-3} \text{ m}^3}{1.85\times10^{22}}$$
$$= 54 \times 10^{-27} \text{ m}^3$$

$\therefore$ Average distance between molecules,

$$= (54 \times 10^{-27})^{1/3}$$
$$= 3.78 \times 10^{-9} \text{ m}$$

38. The molar heat capacity of a gas, C = 42.5 J mol^{-1} K^{-1}, in the process, PT = constant. Find the number of degrees of freedom of molecules in the gas (R = 8.3 J mol^{-1} K^{-1}).

Ans. The equation of gas undergoes the process, given by,

$$PT = \text{constant} \quad \text{......(i)}$$

For 1 mole of an ideal gas,

We have $\quad PV = RT$

$$P = \frac{RT}{V}$$

Where, R is a constant.

Combining it with equation (i).

$$\frac{T^2}{V} = \text{constant}$$

Differentiating: $\dfrac{2TdT}{V} - \dfrac{T^2dV}{V^2} = 0$

Or $\quad \dfrac{dV}{V} = 2\dfrac{dt}{V^2} \quad$ ......(ii)

Now, from the first law of thermodynamics, we have

$$dQ = dU + dW$$

But, $dQ = CdT$ (where C is molar heat capacity of the gas)

$$dU = CVdT$$

And $dW = PdV = RT\, dV/V = 2Rdt$, by equation (ii),

$\therefore$
$$CdT = C_V\, dT + 2RdT$$
$$C = C_V + 2R$$
$$C_V = C - 2R$$

But, C = 42.5 J mol^{-1} K^{-1}(given) $= \dfrac{42.5}{83}R$

(because R = 8.3 J mol^{-1} k^{-1}) = 5R

$\therefore \quad C_V = 5R - 2R = 3R$

From kinetic theory and law of equipment of energy,

$$C_V = \frac{f}{2}R$$

Where, f is the number of degrees of freedom of molecules of the gas.

Comparing the last two expressions.

We have, $\quad f = 6$

NUMERICAL Type Questions

39. A vessel is filled with a gas at a pressure of 76 cm of mercury at a certain temperature. The mass of the gas is increased by 50% by introducing more gas in the vessel at the same temperature. Find out the resultant pressure of the gas.

[Delhi Gov. QB 2022](2m)

Ans. Pressure exerted by a gas,

$$P = \frac{1}{2}\frac{M}{V}v_{rms}^2$$

Since, the temperature T is kept constant, v_{av}^2 and V are also constant.

$\therefore \quad P \propto M$ or $\dfrac{P_2}{P_1} = \dfrac{M_2}{M_1}$

According to the question,

$$\therefore \quad \frac{P_2}{76} = \frac{\left[M_1 + \left(\frac{50}{100}\right)M_2\right]}{M_1} = \frac{3}{2}$$

$$\Rightarrow \quad P_2 = \frac{3}{2} \times 76$$
$$= 114 \text{ cm of mercury.}$$

40. From any given device, the diffusion rate of hydrogen gas averages 28.7 cm^3 s^{-1}, while the diffusion rate of another gas is 7.2 cm^3 s^{-1}. Identify the gas. (3m)

Ans. The diffusion rate of a gas is directly proportional to the rms velocity of the molecule of the gas, *i.e.*

$$R \propto v_{rms}$$

If suffixes 1 and 2 stand for hydrogen and the unknown gas respectively, then we have

$$\frac{R_1}{R_2} = \frac{v_{1rms}}{v_{2rms}} = \sqrt{\frac{M_2}{M_1}}$$

$$\left[\because v_{rms} = \sqrt{\frac{3\,RT}{M}}\right]$$

Where, M_1 and M_2 are molecular masses of hydrogen and the unknown gas respectively.

Here, $\quad R_1 = 28.7 \text{ cm}^3 \text{ s}^{-1}$
$\quad R_2 = 7.2 \text{ cm}^3 \text{ s}^{-1}$

$$\therefore \quad \frac{M_1}{M_2} = \frac{R_1^2}{R_2^2}$$

$$= \frac{(28.7)^2}{(7.2)^2} = 16$$

Or $\quad M_2 = 16M_1$

Now, $M_1 = 2$ (molecular mass of hydrogen)

$\therefore \quad M_2 = 32$

Hence, the unknown gas is Oxygen.

41. **The defective barometer reads 70 cm because there is air above the mercury in a volume having 20 m^3. When the barometer tube is submerged. Because there is too much mercury, the amount of air in the tube above the mercury is reduced to 5 m^3 and the barometer reads 55 cm. What is the correct air pressure of the atmosphere? (3m)**

Ans. If initially, the pressure of air above mercury be P_1 and volume V_1 and after passing the tube in mercury, the air pressure above mercury be P_2 and volume V_2,

Then by the Boyle's law,

$$P_1 V_1 = P_2 V_2$$

Let the correct atmospheric pressure be P cm (of mercury) then,

$$P_1 = P - 70 \text{ cm}$$

and $$P_2 = P - 55 \text{ cm}$$

Substituting values:

$$(P - 70 \text{ cm}) \times 20 \text{ cm}^3 = (P - 55 \text{ cm}) \times 5 \text{ cm}^3$$

or $$P = 75 \text{ cm (Mercury)}.$$

42. **An oxygen cylinder of volume 30 litres has an initial gauge pressure of 15 atmosphere and a temperature of 27°C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atmosphere and its temperature drop to 17°C. Estimate the mass of oxygen taken out of the cylinder.**

(R = 8.31 J/mol^{-1} K^{-1}, Molecular mass of O_2 = 32) [Delhi Gov. QB 2022](5m)

Ans. Volume of oxygen,

$$V_1 = 30 \text{ litres} = 30 \times 10^{-3} m^3$$

Gauge pressure,

$$P_1 = 15 \text{ atm} = 15 \times 1.013 \times 10^5 \text{ Pa}$$

Temperature, $T_1 = 27°C = 300$ K

Universal gas constant,

$$R = 8.314 \text{ J mol}^{-1} K^{-1}$$

Let the initial number of moles of oxygen gas in the cylinder be n_1.

The gas equation is given as:

$$P_1V_1 = nRT_1$$

$$\therefore \quad n_1 = \frac{P_1V_1}{RT_1} = \frac{(15.195 \times 10^5 \times 30 \times 10^{-3})}{8.314 \times 300} = 18.276$$

But, $$n_1 = \frac{m_1}{M}$$

Where,

m_1 = Initial mass of oxygen

M = Molecular mass of oxygen = 32 g

$$\therefore \quad m_1 = N_1M = 18.276 \times 32 = 584.84 \text{ g}$$

After some oxygen is withdrawn from the cylinder, the pressure and temperature reduce.

Volume, $V_2 = 30$ litres $= 30 \times 10^{-3} m^3$

Gauge pressure, $P_2 = 11$ atm $= 11 \times 1.013 \times 10^5$ Pa

Temperature, $T_2 = 17°C = 290$ K

Let n_2 be the number of moles of oxygen left in the cylinder.

The gas equation is given as:

$$P_2V_2 = n_2RT_2$$

$$\therefore \quad n_2 = \frac{P_2V_2}{RT_2} = \frac{(11.143 \times 10^5 \times 30 \times 10^{-30})}{(8.314 \times 290)} = 13.86$$

But, $$n_2 = \frac{m_2}{M}$$

Where,

m_2 is the mass of oxygen remaining in the cylinder

$$\therefore \quad m_2 = n_2 \times M = 13.86 \times 32 = 453.1 \text{ g}$$

The mass of oxygen taken out of the cylinder is given by the relation:

Initial mass of oxygen in the cylinder – Final mass of oxygen in the cylinder

$$= m_1 - m_2 = 584.84 \text{ g} - 453.1 \text{ g} = 131.74 \text{ g} = 0.131 \text{ kg}$$

Therefore, 0.131 kg of oxygen is taken out of the cylinder.